The Materials Science Institute Of Barcelona - Institut de ciencia de materials de Barcelona (ICMAB), UAB

intership report

Magneto-optical Kerr effect measurements at low temperatures

Ondřej Vlašín

September 5, 2008

Abstract

Ellipsometry is particularly attractive for its suitability for *in-situ* measurements and remarkable sensitivity to minute inter-facial effects, such as the formation of sub-monolayer of atoms or magneto-optical (MO) effect. MO methods have been established as a common technique for studying thin film and surface magnetism.
Ellipsometric setups with a photo-elastic modulator are favored for its high signal to noise ratio.
One of the objectives of this work was to calculate the posibility of magneto-optical Kerr effect measurements with all optical devices in one branch of the setup fixed. This would allow for better noise cancelation in fixed branch.

Common way of obtaining equations for a data analysis is a direct derivation from Jones or Mueller matrices of the system for each way of measurement separately. In spite of their various advantages the general scalar equations of the setup are not being used.
General equations allow easy calculation of each component's imperfection influence on the results as well as quick derivation of all ways of measurement possible.
Minor alteration of Jones formalism is proposed here that takes further advantage of the symmetry and allows for easier derivation of general analytical equations for the first and the second harmonic and DC signal.

Novel nulling method for MOKE measurements is calculated and experimentaly prooved that offers better precision in desired region of measurment and reduces moving parts to a single branch of measurement setup. The only trade-off of the proposed method is slightly more complicated nulling procedure.

Furthermore an automatic calibration of spectral magneto-optical Kerr effect measurement setup was designed and applied.

Mathematical Formulation

Each optical device is being assigned Jones 2×2 matrix as follows

$$\mathbb{A} = \mathbb{P} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbb{C}(\gamma) = \begin{bmatrix} 1 & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\gamma} \end{bmatrix} \tag{0.1}$$

$$\mathbb{M}(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\varphi} \end{bmatrix} \tag{0.2}$$

where γ is the retardation angle of the compensator and $\varphi = \varphi_0 + \varphi_A \sin\omega\tau$ is the modulation depth of the photo-elastic modulator (PEM) with the modulation frequency ω, amplitude φ_A and DC signal φ_0. $\mathbb{P}$ and $\mathbb{A}$ stand for polarizer and analyzer.
The rotation of the components is described by the rotation matrix

$$\mathbb{R}_a = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}, \tag{0.3}$$

so that the analyzer rotated at the azimuth angle a

$$\mathbb{A}_a = \mathbb{R}_{-a}\mathbb{A}\mathbb{R}_a = \cos^2 a \begin{bmatrix} 1 & \tan a \\ \tan a & \tan^2 a \end{bmatrix}. \tag{0.4}$$

Proposed Simplification

To take the most advantage of the symmetry, both compensator and modulator can be envisioned as a pair of simultaneous polarizers, where the second polarizer is shifted by $\frac{\pi}{2}$ in azimuth and introduces the modulation to the signal.

$$\begin{aligned} \mathbb{M}_{\varphi m} = &\begin{bmatrix} \cos^2 m & \cos m \sin m \\ \cos m \sin m & \sin^2 m \end{bmatrix} \\ &+ \mathrm{e}^{-\mathrm{i}\varphi} \begin{bmatrix} \sin^2 m & -\cos m \sin m \\ -\cos m \sin m & \cos^2 m \end{bmatrix} \\ &= \mathbb{A}_m + \mathrm{e}^{-\mathrm{i}\varphi}\mathbb{A}_{m+\pi/2} \end{aligned} \tag{0.5}$$

Having done this, the scalar equation of the system can be derived using few simple mathematical operations:

$$\mathbb{P}_a \mathbb{S} \mathbb{P}_b = (S \cdot G_{ab})\mathbb{G}_{ab} \tag{0.6}$$

$$\mathbb{G}_{ab} = \begin{bmatrix} \cos a \cos b & \sin b \cos a \\ \sin a \cos b & \sin b \sin a \end{bmatrix} \tag{0.7}$$

$$\mathbb{A}\mathbb{R}_a\mathbb{G}_{cm}\mathbb{R}_{-p}\mathbb{P} = \cos(a-c)\cdot\cos(m-p)\mathbb{P} \tag{0.8}$$

Sample Representation

The sample can be represented either using the Fresnel coefficients or the complex Kerr effects for *s*-polarized incident beam (χ_s) and for *p*-polarized incident beam (χ_p) and transverse complex Kerr effect χ_t:

$$\mathbb{S} = \begin{bmatrix} r_{ss} & r_{ps}(M_P, M_L) \\ r_{sp}(M_P, M_L) & r_{pp}(M_T) \end{bmatrix} \tag{0.9}$$

$$= r_{ss} \begin{bmatrix} 1 & \chi_s \\ -\chi_p\rho' & \rho' \end{bmatrix} \tag{0.10}$$

where $M_{P,L,T}$ stands for polar, longitudinal and transversal magnetization and where the complex Kerr effects are defined as

$$\chi_s = \frac{r_{ps}}{r_{ss}} = \theta_s + \mathrm{i}\epsilon_s \tag{0.11}$$

$$\chi_p = -\frac{r_{sp}}{r_{pp}} = \theta_p + \mathrm{i}\epsilon_p \tag{0.12}$$

$$\chi_t = \theta_t + \mathrm{i}\epsilon_t = 2\delta\psi \csc 2\psi + \mathrm{i}\delta\Delta, \tag{0.13}$$

where θ is Kerr rotation and ϵThe transverse effect is defined as a small perturbation of ellipsometric angles ψ and Δ with following approximations:

$$\exp\left[\mathrm{i}(\Delta + \delta\Delta)\right] \approx \exp[\mathrm{i}\Delta]\,(1 + \mathrm{i}\delta\Delta) \tag{0.14}$$

$$\tan(\psi + \delta\psi) \approx \tan\psi(1 + 2\delta\psi \csc 2\psi). \tag{0.15}$$

With $\rho = \frac{r_{pp}}{r_{ss}} = \tan\psi \mathrm{e}^{-\mathrm{i}\Delta}$ we can write

$$\rho' = \rho\,(1 + \chi_t) = \tan\psi\,[\Delta_{\Re} + \mathrm{i}\Delta_{\Im}]\,, \tag{0.16}$$

where

$$\Delta_{\Re} = \cos\Delta\,(1 + \theta_t) - \epsilon_t \sin\Delta \tag{0.17}$$

$$\Delta_{\Im} = \epsilon_t \cos\Delta + \sin\Delta\,(1 + \theta_t) \tag{0.18}$$

MOKE measurements with PMSCA ellipsometric setup

Non-depolarizing PMSCA setupcan be fully described by sequence of Jones matrices

$$\mathbf{E}_{\mathrm{out}} = \mathbb{A}\mathbb{R}_a\mathbb{R}_{-c}\mathbb{C}_\gamma\mathbb{R}_c\mathbb{S}\mathbb{R}_{-m}\mathbb{M}_\varphi\mathbb{R}_m\mathbb{R}_{-p}\mathbb{P}\,\mathbf{E}_{\mathrm{in}}. \tag{0.19}$$

Using 0.6 the above equation becomes

$$\begin{aligned}
\mathbf{E}_{\mathrm{out}} &= \mathbb{A}\mathbb{R}_a(\mathbb{A}_c + \mathrm{e}^{-\mathrm{i}\gamma}\mathbb{A}_{c'})\mathbb{S}(\mathbb{A}_m + \mathrm{e}^{-\mathrm{i}\varphi}\mathbb{A}_{m'})\mathbb{R}_{-p}\mathbb{P}\,\mathbf{E}_{\mathrm{in}} \\
&= \mathbb{A}\mathbb{R}_a[(S\cdot G_{cm})\mathbb{G}_{cm} + (S\cdot G_{c'm})\mathbb{G}_{c'm}\mathrm{e}^{-\mathrm{i}\gamma} \\
&\quad +(S\cdot G_{cm'})\mathbb{G}_{cm'}\mathrm{e}^{-\mathrm{i}\varphi} \\
&\quad +(S\cdot G_{c'm'})\mathbb{G}_{c'm'}\mathrm{e}^{-\mathrm{i}\gamma}\mathrm{e}^{-\mathrm{i}\varphi}]\mathbb{R}_{-p}\mathbb{P}\,\mathbf{E}_{\mathrm{in}}
\end{aligned}$$

If we now make use of 0.8 and split the equation into modulated and unmodulated part

$$\mathbf{E}_{\mathrm{out}} = \begin{bmatrix} \Gamma_0 + \mathrm{e}^{-\mathrm{i}\varphi}\Gamma \\ 0 \end{bmatrix} \mathbf{E}_{\mathrm{in}}$$

where

$$\begin{aligned}
\Gamma_0 = &\quad (S\cdot G_{cm})\cos(a-c)\cos(p-m) \\
&+ (S\cdot G_{c'm})\sin(a-c)\cos(p-m)\mathrm{e}^{-\mathrm{i}\gamma} \\
\Gamma = &\quad (S\cdot G_{cm'})\cos(a-c)\sin(p-m) \\
&+ (S\cdot G_{c'm'})\sin(a-c)\sin(p-m)\mathrm{e}^{-\mathrm{i}\gamma}
\end{aligned}$$

the intensity of the field can be calculated as

$$\begin{aligned}
I &= \mathbf{E}^\dagger\mathbf{E} = (\Gamma_0 + \mathrm{e}^{-\mathrm{i}\varphi}\Gamma)^*(\Gamma_0 + \mathrm{e}^{-\mathrm{i}\varphi}\Gamma) \\
&= |\Gamma_0|^2 + |\Gamma|^2 + 2\cos\varphi\Re\{\Gamma\Gamma_0^*\} \qquad (0.20) \\
&\quad +2\sin\varphi\Im\{\Gamma\Gamma_0^*\} \\
&= |\Gamma_0|^2 + |\Gamma|^2 + 2\cos\varphi\Re\{\Gamma_0\Gamma^*\} \qquad (0.21) \\
&\quad -2\sin\varphi\Im\{\Gamma_0\Gamma^*\} \\
&= I_0 + \cos\varphi I_c + \sin\varphi I_s
\end{aligned}$$

and using Bessel functions expansion of $\sin\varphi$ and $\cos\varphi$

$$\sin\varphi = \mathrm{J}_0(\varphi_A)\sin\varphi_0 + 2\mathrm{J}_1(\varphi_A)\cos\varphi_0\sin\omega\tau \tag{0.22}$$
$$+2\mathrm{J}_2(\varphi_A)\sin\varphi_0\cos 2\omega\tau + \ldots$$

$$\cos\varphi = \mathrm{J}_0(\varphi_A)\cos\varphi_0 - 2\mathrm{J}_1(\varphi_A)\sin\varphi_0\sin\omega\tau \tag{0.23}$$
$$+2\mathrm{J}_2(\varphi_A)\cos\varphi_0\cos 2\omega\tau + \ldots$$

we arrive at the general expressions for the first and the second harmonic of the PMSCA ellipsometric setup:

$$I_\omega = -4\mathrm{J}_1(\varphi_A)\,(\cos\varphi_0\Im\{\Gamma_0\Gamma^*\} + \sin\varphi_0\Re\{\Gamma_0\Gamma^*\}) \tag{0.24}$$

$$I_{2\omega} = 4\mathrm{J}_2(\varphi_A)\,(\cos\varphi_0\Re\{\Gamma_0\Gamma^*\} - \sin\varphi_0\Im\{\Gamma_0\Gamma^*\}) \tag{0.25}$$

For dc signal we can write:

$$I_{dc} = |\Gamma_0|^2 + |\Gamma|^2 + 4\mathrm{J}_0(\varphi_A) \tag{0.26}$$
$$(\cos\varphi_0\Re\{\Gamma_0\Gamma^*\} - \sin\varphi_0\Im\{\Gamma_0\Gamma^*\})$$

According to equation 0.21 and 0.20:

$$I_s = 2\Im\{\Gamma\Gamma_0^*\} = -2\Im\{\Gamma^*\Gamma_0\} \tag{0.27}$$

$$I_c = 2\Re\{\Gamma\Gamma_0^*\} = 2\Re\{\Gamma^*\Gamma_0\} \tag{0.28}$$

$$I_0 = |\Gamma_0|^2 + |\Gamma|^2 \tag{0.29}$$

To obtain equations for the first and the second harmonic signal intensity it is sufficient to calculate $\Gamma\Gamma_0^*$. For short and for better readability I'm going to use c_a, s_a, t_a instead of $\cos a, \sin a, \tan a$ where convenient.

$$\begin{aligned}
\frac{\Gamma\Gamma_0^*}{\frac{1}{2}|r_{ss}|^2 s_{2(p-m)}} = &\ (1-\Omega_0)\left(\theta_s c_{2m} - \tfrac{1}{2}s_{2m} + \mathrm{i}\epsilon_s\right) \\
&-\Omega_0 t_\psi^2\left(\theta_p c_{2m} - \tfrac{1}{2}s_{2m} + \mathrm{i}\epsilon_p\right) \\
&+(\Omega_1 - \mathrm{i}\Omega_2)t_\psi\left(\tfrac{1}{2}s_{2m}(X_1 + \mathrm{i}X_2) + c_m^2(\Delta_{\Re} + \mathrm{i}\Delta_{\Im})\right) \\
&+(\Omega_1 + \mathrm{i}\Omega_2)t_\psi\left(\tfrac{1}{2}s_{2m}(X_1 + \mathrm{i}X_2) - s_m^2(\Delta_{\Re} - \mathrm{i}\Delta_{\Im})\right)
\end{aligned}$$

where

$$\begin{aligned}
\Omega_0 &= s_{a-c}^2 c_c^2 + c_{a-c}^2 s_c^2 + \frac{1}{2}s_{2(a-c)}s_{2c}c_\gamma \\
2\Omega_1 &= c_{2(a-c)}s_{2c} + c_{2c}s_{2(a-c)}c_\gamma \\
2\Omega_2 &= s_{2(a-c)}s_\gamma
\end{aligned}$$

and

$$\begin{aligned} X_1 &= \Delta_{\Re}(\theta_p+\theta_s)+\Delta_{\Im}(\epsilon_s-\epsilon_p) \\ X_2 &= \Delta_{\Im}(\theta_p+\theta_s)-\Delta_{\Re}(\epsilon_s-\epsilon_p). \end{aligned}$$

According to 0.27 and 0.28

$$\begin{aligned} \frac{I_s}{|r_{ss}|^2 s_{2(p-m)}} &= t_\psi(\Omega_1\Delta_{\Im}-\Omega_2\Delta_{\Re}) && (0.30) \\ & -\Omega_0(\epsilon_s+t_\psi^2\epsilon_p)+\epsilon_s \\ & +s_{2m}\Omega_1 t_\psi X_2 \\ \frac{I_c}{|r_{ss}|^2 s_{2(p-m)}} &= [t_\psi(\Omega_1\Delta_{\Re}+\Omega_2\Delta_{\Im}) && (0.31) \\ & -\Omega_0(\theta_s+t_\psi^2\theta_p)+\theta_s]c_{2m} \\ & +s_{2m}\Omega_1 t_\psi X_1+s_{2m}[\Omega_0 c_\psi^{-2}-1] \end{aligned}$$

If we substitute into equation 0.29:

$$\frac{I_0}{\frac{1}{2}|r_{ss}|^2} = \Omega_0 t_\psi^2+(\Omega_1X_1+\Omega_2X_2)t_\psi+1-\Omega_0$$

Obtaining the first and the second harmonic zero

Without magnetic field ($\Delta_{\Re}=\cos\Delta$, $\Delta_{\Im}=\sin\Delta$) we get for the zero of the first harmonic ($I_\omega \approx I_s = 0$)

$$\begin{aligned} \cot\Delta &= \frac{\Omega_1}{\Omega_2} = \cot 2(a-c)\sin 2c\csc\gamma && (0.32) \\ & +\cos 2c\cot\gamma, \end{aligned}$$

and for the second harmonic zero ($I_{2\omega} \approx I_c = 0$)

$$\tan\psi = \pm\sqrt{\alpha^2-1+\Omega_0^{-1}}-\alpha \qquad (0.33)$$

$$\alpha = \tfrac{1}{2}\cot 2m\,.\,(\Omega_1\cos\Delta+\Omega_2\sin\Delta)\Omega_0^{-1}. \qquad (0.34)$$

Here are some measurment configuration as proposed by [1]:

Fixed compensator

Plugging $c=\pm\pi/4$ and $\gamma=\frac{\pi}{2}$ into 0.32 we obtain

$$\cot\Delta = (\mp)_c\cot\left(2a(\pm)_c\frac{\pi}{2}\right).$$

Furthermore, using equality

$$\cot^{-1}(a-b) = \cot^{-1}a+\cot^{-1}[(a^2-ab+1)/b]$$

and noting that $\cos(\gamma-\frac{\pi}{2}) \gg \sin\gamma$, as $\gamma\to\frac{\pi}{2}$ we can write

$$\Delta = (\pm)_\Delta(\mp)_c\left(2a(\pm)_c\frac{\pi}{2}\right)\pm\left[\cos^2(2a)\,\delta_c\delta_\gamma\right],$$

where $(\pm)_\Delta$ corresponds to periodicity of cotangent.

Imprecision in calculation of Δ can be approximated as

$$\delta\Delta = \delta_a+\delta_c+\cos^2(2a)\delta_c\delta_\gamma,$$

where $\delta_c=\frac{\pi}{2}-2c$ and $\delta_\gamma=\frac{\pi}{2}-\gamma$. δ_γ contains imperfections of the quarter-wave plate and varies with wavelength as $\gamma=\frac{2\pi}{\lambda}|n_e-n_0|d$. For example, effect of using $650nm$ laser with a first order quarter wave plate made for $632.8nm$ laser ($\delta\gamma=0.04$) is still negligible. Imprecision in calculation of Δ angle is therefore double the sum of the smallest adjustable azimuth step of compensator and of analyzer. In basic optical devices this step is in the order of 0.01, commercial ellipsometres go to precisions around 1 mrad for azimuth angles.

Note that the first harmonic signal is independent of modulator azimuth and with the modulator azimuth fixed to $\pm\frac{\pi}{4}$, the second harmonic is independent of Δ.

General equations 0.30 and 0.31 collapse with this configuration to standard pair of equations [2]

$$\begin{aligned} I_s &= \tan\psi(\sin 2a\sin\Delta\pm\cos 2a\cos\Delta\sin\gamma) \\ I_c &= \tan\psi\cos 2m(\sin 2a\cos\Delta\mp\cos 2a\sin\Delta\sin\gamma) \quad (0.35) \\ &+ \tfrac{1}{2}\sin 2m\left[(1+\tan^2\psi)(\cos 2a\cos\gamma)+(1-\tan^2\psi)\right]. \end{aligned}$$

Plugging 0.32 into 0.35 we get

$$\begin{aligned} \tan\psi &= (\pm)_\Delta\frac{1}{\tan 2m}(\pm)_\psi\frac{1}{\sin 2m} && (0.36) \\ &= \left\{ \begin{array}{cc|l} \Delta & \psi & \\ \hline + & + & \psi=-m+\pi/2 \\ + & - & \psi=-m \\ - & + & \psi=+m \\ - & - & \psi=+m-\pi/2 \end{array}\right. \end{aligned}$$

where $(\pm)_\psi$ corresponds to two roots of the quadratic equation.

The second harmonic signal

can be as well nulled by rotating analyzer and direct relation of analyzer azimuth and ψ can be obtained in the form

$$\begin{aligned} \tan\psi_{1,2} &= \frac{-\sin 2a\cos\Delta\pm\cos 2a(\pm)_{1,2}\sqrt{D}}{2\sin^2 a\pm\cos 2a} \\ D &= \sin^2 2a(\cos^2\Delta\mp 2+1)\mp\sin 4a\cos\Delta\pm 2. \end{aligned}$$

where compensator is fixed at $\pm\pi/4$, PEM-polarizer system at $\pi/8$ and retardation is assumed $\pi/2$.

Fixed analyzer

Similar results can be obtained for fixed analyzer and rotating compensator.

$$
\begin{aligned}
a = 0, \pm\tfrac{\pi}{2}; I_\omega = 0 \quad &\Rightarrow \quad \tan\Delta = \sec 2c \\
a = \pm\tfrac{\pi}{4}; I_\omega = 0 \quad &\Rightarrow \quad \cot\Delta = \pm\sec 2c \mp \cos 2c \\
a = \pm\tfrac{\pi}{4}; I_{2\omega} = 0 \quad &\Rightarrow
\end{aligned}
$$

$$
\begin{aligned}
t_\psi \quad = \quad & \frac{(\pm)_a c_{2m} s_\Delta c_{2c} (s_{2c}^2 t_{2c}^2 + 1)}{1(\mp)_a \frac{1}{2} s_{4c} s_{2m}} \qquad (0.37) \\
& \left[-1(\pm)_\psi \sqrt{1 - \frac{t_{2m}^2}{s_\Delta^2} \frac{3(\mp)_a 2 s_{4c} + \frac{1}{4} s_{4c}^2}{c_{2c}^2 (s_{2c}^2 t_{2c}^2 + 1)^2}} \right]
\end{aligned}
$$

Fixed modulator-polarizer

For the simplest configuration without compensator and with polarizer at 45° and modulator at 0°, the above equations collapse to

$$I_{a=0} = |r_{ss}|^2 (1 + \theta_s \cos\varphi + \epsilon_s \sin\varphi) \qquad (0.38)$$

$$I_{a=\frac{\pi}{2}} = |r_{pp}|^2 (1 - \theta_p \cos\varphi + \epsilon_p \sin\varphi) \qquad (0.39)$$

If we have modulator azimuth fixed to $\pm\frac{\pi}{4}$ and polarizer azimuth to 0 we obtain for the first and the second harmonic zeros

$$I_\omega \approx I_s = 0: \ \tan\Delta = \frac{\Omega_2}{\Omega_1} \qquad (0.40)$$

$$I_{2\omega} \approx I_c = 0: \ \cos^2\psi = \Omega_0, \qquad (0.41)$$

or in another words

$$\cot\Delta = \cot 2b \sin 2c \csc\gamma + \cos 2c \cot\gamma \qquad (0.42)$$

$$\cos^2\psi = \sin^2 b \cos^2 c + \cos^2 b \sin^2 c - \frac{1}{2} \sin 2b \sin 2c \cos\gamma, \qquad (0.43)$$

where $b = c - a$. The above equations can be envisioned in $\psi\Delta$ plane as a contours of fixed compensator azimuth and contours of fixed relative angle between compensator and analyzer.

With an ideal quarter wave plate ($\gamma = \frac{\pi}{2}$)

$$\cot\Delta = \cot 2b \sin 2c \qquad (0.44)$$

$$\cos^2\psi = \sin^2 b \cos^2 c + \cos^2 b \sin^2 c. \qquad (0.45)$$

In the set-up with fixed compensator, linear relations between analyzer and polarizer azimuth and the ellipsometric angles were obtained. This is not the case with the fixed polarizer setup. If we plot ψ against Δ, compensator azimuth corresponds to rotation about the origin at $\Delta = \frac{\pi}{2}, \psi = \frac{\pi}{4}$ and relative angle between analyzer and compensator corresponds to the distance from the origin.

Variable retardation angle

By fixing modulator at $\pm\frac{\pi}{4}$ we can make the second harmonic independent on Δ, and with analyzer fixed at $\pm\frac{\pi}{4}$ and compensator at $\pm\frac{\pi}{8}$ we obtain

$$I_\omega = 0: \ \tan\Delta = \frac{\sqrt{2}\sin\gamma}{1 + \cos\gamma}$$

$$I_{2\omega} = 0: \ \tan\psi = \left(\frac{1 - 3\cos\gamma}{1 + \cos\gamma} \right)^{(\pm)_c (\pm)_a \frac{1}{2}}$$

which shows that ellipsometric angles ψ and Δ can also be obtained by alternating the retardation angle of compensator using for example Babinet-Soleil compensator.

Magneto-optical Kerr effects measurement

At the null position for both the first and the second harmonic signal equations for the first and the second harmonic reduce to

$$
\begin{aligned}
\frac{I_s}{|r_{ss}|^2 s_{2(p-m)}} &= t_\psi (\Omega_1 (c_\Delta \epsilon_t + s_\Delta \theta_t) - \Omega_2 (c_\Delta \theta_t - s_\Delta \epsilon_t)) \\
&\quad - \Omega_0 (\epsilon_s + t_\psi^2 \epsilon_p) + \epsilon_s + s_{2m} \Omega_1 t_\psi X_2 \\
\frac{I_c}{|r_{ss}|^2 s_{2(p-m)}} &= [t_\psi (\Omega_1 (c_\Delta \theta_t - s_\Delta \epsilon_t) + \Omega_2 (c_\Delta \epsilon_t + s_\Delta \theta_t)) \\
&\quad - \Omega_0 (\theta_s + t_\psi^2 \theta_p) + \theta_s] c_{2m} + s_{2m} \Omega_1 t_\psi X_1.
\end{aligned}
$$

Thanks to odd parity of sine, the mixed-Kerr-effects coefficients X can be elliminated by zone averaging [3].

Fixed compensator

With compensator fixed at 45° and with positive values of analyzer and averaging over two zones with $\pm$ modulator azimuth the above equations further reduce to

$$
\begin{aligned}
\frac{I_s}{|r_{ss}|^2 \sin 2(p-m)} &= \frac{1}{2}\epsilon_s + \epsilon_t \tan\psi - \frac{1}{2}\epsilon_p \tan^2\psi \\
\frac{I_c}{|r_{ss}|^2 \sin 2(p-m) \cos 2m} &= \frac{1}{2}\theta_s + \theta_t \tan\psi - \frac{1}{2}\theta_p \tan^2\psi
\end{aligned}
$$

Calibration

Theory

Several approaches are possible for system calibration. Calibration methods using osciloscope, are fast but limited in precision and very demanding on frequency stability and therefore couldn't be used with ICMAB setup. Alternative approach is based on Bessel functions.

Rotating analyzer

For polarizer fixed at 45° , modulator at 0 and rotating analyzer we get

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\cos a & -\sin a\\ \sin a & \cos a\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \mathrm{e}^{-\mathrm{i}\varphi}\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}.$$

The intensity on the detector can be calculated as

$$I = \frac{1}{2}\left(1 - \sin 2a\cos\varphi\right).$$

Expanding cosine with 0.23

$$\begin{aligned} 2I &= 1 - \sin 2a && (0.46)\\ & [\mathrm{J}_0(\varphi_A)\cos\varphi_0 - 2\mathrm{J}_1(\varphi_A)\sin\varphi_0\sin\omega\tau \\ & +2\mathrm{J}_2(\varphi_A)\cos\varphi_0\cos 2\omega\tau] && (0.47)\end{aligned}$$

The relation between ac and dc components can be obtained in the form:

$$\frac{I_{2\omega}}{I_0} = \frac{-2J_2(\varphi_A)\sin 2a\cos\varphi_0}{1 - J_0(\varphi_A)\sin 2a\cos\varphi_0}\cdot k$$

$$\frac{I_\omega}{I_0} = \frac{2J_1(\varphi_A)\sin 2a\sin\varphi_0}{1 - J_0(\varphi_A)\sin 2a\cos\varphi_0}$$

Setting the PEM retardation angle to the first zero of the Bessel function of the first kind ($\varphi_A = 2.4048$) we can write

$$\frac{I_{2\omega}}{I_0} = 2k\cdot J_2(\varphi_A)\sin 2a\cos\varphi_0.$$

To check for the linearity of the first harmonic, we can write

$$\tan\varphi_0 = \frac{I_\omega}{I_{2\omega}}\frac{J_2(\varphi_A)}{J_1(\varphi_A)}$$

for the *residual birefringence* of the PEM. Using this method residual birefringence of PEM used with MOKE setup at ICMAB was found to be approximately 0.03 rad.

For setup used at ICMAB the **k** coefficient was found out to be 0.071deg^{-1} with 650 nm laser. This result in the limits of measurement precision corresponds to the theoretical value of ratio between absolute (dc) and effective (ac) voltage ($\sqrt{2}$).

Rotating modulator

The Jones matrix of light incident on detector for analyzer fixed at 0 is in the form

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\cos m & -\sin m\\ \sin m & \cos m\end{bmatrix} \qquad (0.48)$$

$$\begin{bmatrix}1 & 0\\ 0 & \mathrm{e}^{-\mathrm{i}\varphi}\end{bmatrix}\begin{bmatrix}\cos m & \sin m\\ -\sin m & \cos m\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$$

The intensity on the detector can be calculated as

$$I = \frac{1}{2}\left(1 + \frac{1}{2}\sin 4m[1+\cos\varphi]\right). \qquad (0.49)$$

For small azimuths of modulator $\sin m \to m$ and we can write

$$2I = 1 + 2m(1+\cos\varphi) \qquad (0.50)$$

Using Bessel function expansion of cosine and $J(\varphi_A = 2.4048) = 0$, the relation between ac and dc components can be written as

$$\frac{I_{2\omega}}{I_0} = 4kJ_2(\varphi_A)\frac{m}{1+2m}. \qquad (0.51)$$

Proposed fast spectral calibration method

The zero of the first and the second harmonic corresponds to constant dc signal. This condition can by satisfied by rotating analyzer with any arbitrary voltage applied to PEM. If we plot the first and second harmonic signal and dc signal against the voltage applied to PEM at this point, we obtain three constant functions.

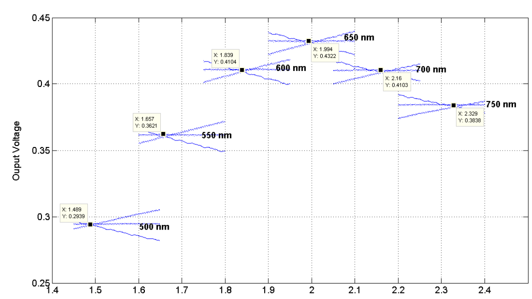

Figure 0.1: Spectral PEM calibration

Then the analyzer is rotated to an arbitrary azimuth (lets say 10 degrees from constant position) and the dc signal over the spectrum of voltage applied to PEM is recorded again. The same measurment is then repeated with oposite relative azimuth of analyzer $(-10°)$. The intersect of the three curves is then interpolated.

Retardation angle corresponding to voltage applied to PEM obtained by this calibration is 2.405 rad because $J_1(2.405) = 0$.

DC and the second harmonic voltage are then recorded while rotating analyzer back to zero to obtain the calibration parameter k' from the equation 0.51. Parameter k'should be constant over the whole spectrum and serves as a control of the calibration stability.

To speed up the spectral measurement an approximate empiric formula for visible region was found

$$U_{PEM}\left(J_1(2.405) = 0\right) \approx 3.05 \times \lambda[\mu m]\, V, \qquad (0.52)$$

for voltage applied to PEM that corresponds to $J_1(2.405) = 0$. For sufficient precision of interpolation measurement for each wavelength is carried out in the 0.2 V interval centered at the approximate value as in figure .

Bibliography

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